ABSTRACT

In this paper, a 2D transient finite element analysis was carried out for a new type of solar powered injera baking system. In the proposed system, (currently under development), heat transfer oil is heated using solar energy by parabolic trough and the oil circulates through the space below the baking pan in the kitchen.

Based on previous finite element study on existing electric injera baking pans, a new type of baking pan made from ceramic with 8mm thickness was manufactured and used for the proposed system. The model was further extended to study the heat up time and temperature distributions during initial heat up and cyclic baking of the new model. The proposed baking pan that uses solar energy gives acceptable heat up and baking time compared to existing conventional baking methods. Generally, the finite element model predicts well the temperature distributions during initial heat up and cyclic baking.

1. INTRODUCTION

Energy plays an important role on the development of a nation and development is possible through an increasing efficient use and extensive harnessing of various forms of energy. Despite rapid urbanization, the majority of Ethiopians still live in rural areas, and access to and utilization of energy resources varies considerably thorough the country. Even though Ethiopia has enormous potential for developing various energy resources, the per capita energy consumption remains to be among the lowest in the world [1].

Injera is spongy flat bread with a distinctive test and texture. It is predominantly eaten as staple food item in Ethiopia and some parts of East Africa. It is similar to an Indian Chapatti with small bubbly structures or eyes on top. In most households of Ethiopia, the energy demand for baking injera is largely met with bio-mass such as: fuel wood, agricultural residue and dung cakes. Whereas, electricity is used in some of urban households.

Injera baking is the most energy intensive process. Injera baking requires temperatures ranging from 180° C - 220° C [3]. It is reported that cooking and baking account for over 50% of all primary energy consumption in the country. Introducing a new alternative energy source for baking injera is an important aspect from environmental and economic point of view. Solar powered injera baking can benefit the environment by decreasing deforestation and the associated desertification. It can also decrease the health hazards associated with indoor fire cooking. Moreover, Women in villages and in some urban communities are relieved from economic burdens associated with firewood gathering or purchase.

For a solar cooking system to be accepted and adopted in most of the households, the following objectives have to be satisfied [7].
- The cooking should be done without moving out of the kitchens.
- A reduction in the use of conventional energy.
- Cooking should be carried out at any time of day.
- Time taken for cooking must be comparable with conventional cooking.

In order to satisfy the above objectives, a solar powered injera baking system is proposed wherein the solar energy is transferred to the kitchen by means of a circulating heat transfer fluid. In conventional injera baking, there is a smoke emission from fuel-wood combustion which is the major source of indoor pollution, especially in rural and poor urban communities. This smoke contains pollutants and particulates that adversely affect the health of women. The proposed system is free of smoke, hence, improves the health and safety of the user. The study on this paper focuses on the heat collecting elements (injera and the baking pan), based on previous study on existing electric injera baking pans.

Purlis and Salvadori (2009a) developed a mathematical model for bread baking process. They used experimental data (temperature, water content, weight loss and crust thickness) obtained during baking to understand the simultaneous heat and mass transfer occurring during the process. They recommend using the mathematical model presented in their work to study the baking of other products such as biscuits and cakes.

Mondal and Datta (2010) developed a 2D CFD model for crustless bread baking to facilitate better understanding of the baking process. Simulation was done for heat and mass transfer from the bread during baking. They found that, the core temperature of the bread reached 95°C at the end of baking, where moisture content of the bread complies with good quality bread.

Purlis and Salvadori (2009b) predicted temperature and water content in the bread during baking. Finite element method was used based on a mathematical model considering moving evaporation front, evaporation-condensation mechanism and crust development during baking.

Another suggested hypothesis for porous-bodies is based on mathematical model proposed by Luikov (1975) to describe simultaneous heat and mass transfer during drying and baking. This phenomenological approach applies the concept of irreversible thermodynamics and includes the effect of temperature on the water transport (i.e. thermo diffusion). This model has been used as the basis of this study, but here based on thermo-physical properties of injera batter obtained from its composition (i.e., Carbohydrate, protein, fat, ash, fiber and water contents, temperature and density) [10].

Previous work by Assefa Ayalew (2009) on transient heat transfer analysis of existing electric injera baking pan using finite element method was used as a basis for this work. The study findings show that, major improvements in energy efficiency can be achieved if baking pan thermal conductivity and thickness were improved. Based on these findings, a new ceramic pan with higher thermal conductivity and reduced thickness was manufactured in local ceramic factory and used for the proposed solar powered injera baking system. The finite element model is further extended to study the heat up time and temperature distributions during initial heat up and cyclic baking of the new system.

2. DESCRIPTION OF THE SYSTEM.

The block diagram of the proposed solar powered injera baking system is shown in Fig. 1 [3]. The system consists of: parabolic trough, pumps, heat storage tank and the injera baking pan (‘mitad’). The parabolic trough is used to collect solar energy and increase the temperature of the fluid.

![Block diagram of solar powered injera baking oven](Image)

Fig. 1: Block diagram of solar powered injera baking oven

The heat transfer fluid (Shell Thermia B) coming from the trough mixes with the fluid coming from the baking pan within the heat storage tank. Heat is transferred from the oil to a steel plate then to ceramic pan and finally to injera during baking. The injera baking pan is placed in the kitchen where the baking is done. All other components are placed at intermediate levels according to the building requirements. As mentioned before, the section which was considered for this study is the region comprising only the injera baking pan (mitad).

When the injera-baking pan is considered, the heat transfer is radially symmetric. Hence, the problem can be reduced to 2D axis symmetric heat transfer. The model to be considered is shown below (Fig. 2).
The initial condition for the baking pan can be taken as a uniform temperature which is dependent on position. Hence assuming a uniform temperature of 20 °C on pan surface at t = 0,

\[ T(r, z, 0) = T_{\text{room}} = 20^\circ \text{C} \]  \hspace{1cm} (2)

**Boundary Conditions**

The heat transfer realized by convection and radiation mechanisms defines the boundary condition for the temperature at the top surface of the baking pan. A general set of boundary conditions for the baking pan both on top and bottom surfaces is given by [4]:

\[ k \left( \frac{\partial T}{\partial n} \right)_{r,0,t} + h_c(T - T_a) + h_r(T - T_a) = 0 \]  \hspace{1cm} (3)

The first term \((k \frac{\partial T}{\partial n})\) is the amount of heat passing in to the body, the second term \((h_c(T-T_a))\) and the third term \((h_r(T-T_a))\) are the heat loss from (or to) at the surface depending on the temperature gradient of the surface of baking pan.

At the bottom of the solar baking pan the boundary condition is:

\[ -\left( k \frac{\partial T}{\partial n} \right)_{r,0,t} = h_{c1}(T_s - T_f) \]  \hspace{1cm} (4)

Where: \(T_f\) is surface temperature of the baking pan, \(T_f\) is heating fluid temperature which is oil and \(h_{c1}\) is the convective heat transfer coefficient.

At the top surface, the solar baking pan has convective and radiative boundary conditions due to heat loss to the surrounding air, which is given by:

\[ -\left( k \frac{\partial T}{\partial n} \right)_{r,z,t} = h_{c2} \times (T_s - T_{\infty}) \]  \hspace{1cm} (5)

Where: \(h_{c2} = h_c + h_r\) -is the convective and radiative heat transfer coefficient at the surface of the pan\((W/m^2.K)\).

3.2 Governing Equations for Injera during Baking

The governing equation during injera baking is derived based on Luikov’s heat and mass transfer theory. The effect of phase change on the temperature gradient was considered by neglecting the effect of pressure transfer. The mathematical model consists of coupled partial differential equations for simultaneous heat and mass transfer [5].

The balance of thermal energy with in a capillary-porous body can be described as [4, 5, 10]:

\[ \rho c_p \frac{\partial T}{\partial t} = -div(j_q) - \sum_{j=1}^{3} h_j I_j \]  \hspace{1cm} (6)
In baking practice the heat flux \( j_q \) is normally related to the temperature gradient, which is transfer of heat by conduction given by:

\[
j_q = -k \nabla T = -k \left( \frac{\partial T}{\partial r} n_r + \frac{\partial T}{\partial z} n_z \right)
\]  

(6a)

The term \( \sum_{j=1}^{3} h_I I_j \) is the heat source or sink. The moisture in vapor form is denoted by suffix 1, in liquid form by 2, and in solid form by 3. The source of heat is due to the phase change of water contained within the body. By neglecting the effect of moisture potential and pressure, it can be written as:

\[
\sum_{j=1}^{3} h_I I_j = \varepsilon_r \rho h_{fs} \frac{\partial m}{\partial t}
\]  

(6b)

By introducing equation (6a) and (6b) in (6) gives the energy equation during baking of injera:

\[
\rho c_p \frac{\partial T}{\partial t} = k \nabla^2 T + \varepsilon_r \rho h_{fs} \frac{\partial m}{\partial t}
\]  

(7)

From the mass transfer equation, the mass balance for one of the bounded materials, vapor or liquid, in the capillaries porous bodies not only related to gradient of moisture concentration, but also the temperature gradient.

\[
\frac{\partial m}{\partial t} = -\text{div}[-D \rho (\nabla m + \delta \nabla T)] + I_m
\]  

(8)

Assuming there is no moisture gain during baking and moisture exists as vapor and liquid, \( I_m=0 \). By substituting Eqn. (8) in to Eqn. (7) and neglecting the effect of change of moisture potential, we obtain differential equation for heat transfer in injera:

\[
\frac{\partial T}{\partial t} = \alpha_{eff} \nabla^2 T
\]  

(9)

Where \( \alpha_{eff} = \frac{k + \varepsilon_r h_{fs} \rho \delta \sigma D}{\rho c_p} \) is the effective thermal diffusivity of the medium.

\[ \text{Initial condition} \]

The initial temperature and moisture content within injera before baking is assumed uniform and given by:

\[
T(x, r, t_0) = T_0 = 20^\circ\text{C at } t = 0
\]

\[
m(x, r, t_0) = x_{m0} = 73\% \text{ at } t = 0
\]  

(10)

\[ \text{Boundary Conditions} \]

A general set of boundary conditions for heat and mass transfer for injera during baking is given by:

\[
k \frac{\partial T}{\partial n} + j_q + h_b (T_{x2} - T_x) = 0 \quad \Gamma_2
\]  

(11)

At the top surface of injera assuming 3 mm thickness for injera \( (z = 0.003m \text{ and } \tau > 0) \):

\[
k \frac{\partial T}{\partial n} = h_{x1} (T_x - T) - j_q
\]  

(12)

The boundary condition at the interface of the injera (assuming uniform thickness of injera) and baking pan is given by:

\[
T = T_b(t) \quad \text{For } (z = 0.0m \text{ and } \tau > 0)
\]  

(13)

Where: \( T_b \) is the temperature of phase change from liquid to vapor.

3.3 Development of the Finite Element Equations

The governing equations are transformed into element equations using Galerkin’s Weight Residual method. A three node triangular isoperimetric element has been used as the basic element type. Gauss-Legendre quadrature for numerical integration and the fully implicit or backward difference form of the integration scheme were used.

Using Galerkin’s method the basic governing equation in Finite Element Method (FEM) written as:

\[
\int_{a}^{b} \left[ N_i \left( \frac{\partial}{\partial t} \frac{\partial T}{\partial t} + \frac{\partial T}{\partial t} \right) \right] \text{dQ} = \int_{a}^{b} \left[ \frac{N_i}{k} \frac{Q}{Q} \right] \text{dQ}
\]

(14)

Expressing the temperature by interpolating functions and applying Gauss-Legendre integration scheme, the above expression can be written as:

\[
[C]^{(e)} \{ \dot{T} \} + [K]^{(e)} \{ \dot{T} \} = \{ Q \}^{(e)}
\]

(15)

Applying the boundary conditions mentioned above and using the backward difference method gives the final solution as:

\[
[T]^{t+\Delta t} = [A]^{-1} \{ V \}^{t+\Delta t}
\]

(16)

Where:

\[
[A] = ([C] + \Delta t [K])
\]

\[
[V]^{t+\Delta t} = [C]^{(e)} \{ \dot{T} \}^{t+\Delta t} + \Delta t [Q]^{(e)}^{t+\Delta t}
\]

(17)

Finite element discretization of injera and baking pan used for cyclic baking simulation is shown below (Fig. 3). Nodes 111 – 121, represent the nodes on the upper surface of the baking ban. Nodes 144 – 154, represent the nods on the
upper surface of the injera. During initial heat up, the nodes consisting only the baking pan (nodes 1 – 121) were considered in the simulation. For cyclic baking, nodes representing the injera (nodes 111 – 154) were superimposed on nodes representing the baking pan. Due to symmetry, half of the thickness of the baking pan and injera were considered.

![Finite element discretization of injera and baking pan](image)

Fig. 3: Finite element discretization of injera and baking pan

Summary of thermo-physical and mass properties of injera and baking pan and various parameters used for determining individual heat transfer coefficients used in FEM are given below (Table 1).

<table>
<thead>
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<th>TABLE 1: THERMO-PHYSICAL PROPERTIES OF INJERA AND BAKING PAN</th>
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<td>Density of ceramic pan</td>
</tr>
<tr>
<td>Thickness of ceramic pan</td>
</tr>
<tr>
<td>Diameter of ceramic pan</td>
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<tr>
<td>Average specific heat of injera batter</td>
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<tr>
<td>Thermal conductivity of ceramic pan</td>
</tr>
<tr>
<td>Specific heat of ceramic pan</td>
</tr>
<tr>
<td>Thermal conductivity of injera batter</td>
</tr>
<tr>
<td>Thickness of injera (average)</td>
</tr>
<tr>
<td>Moisture content in injera batter (wet-basis)</td>
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<td>Boiling heat transfer coefficient during baking on pan surface</td>
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<tr>
<td>Free convective heat transfer coefficient over the baking pan(h_c)</td>
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<tr>
<td>Radiative heat transfer coefficient over the baking pan (h_r)</td>
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</tbody>
</table>

4. RESULTS AND DISCUSSION

Temperature variations on surface of the baking pan during initial heat-up and baking cycle were successfully simulated for different oil temperatures from the parabolic trough.

4.1 Heat up time of the proposed solar baking pan

The heat up time of a baking pan is the amount of time required to increase the surface temperature of the pan from its initial temperature (20-25°C) to the temperature required for baking of injera (180-220°C). The heat up time depends on the input oil temperature, thickness and thermal property of the baking pan.

The temperature profile on surface node (node 118) of 0.008 m ceramic baking pan as a function of heat up time for different heated oil input temperatures is shown below (Fig. 4). In order to start baking (to reach surface temperature of around 200°C), it takes approximately 236 and 600 seconds for heated oil temperatures of 325 and 250°C respectively.

![Heat up time of 0.008m ceramic pan for different heated oil temperatures](image)

Fig. 4: Heat up time of 0.008m ceramic pan for different heated oil temperatures.

The required baking temperature is reached as long as the oil temperature at the bottom of the ceramic pan is kept within the required temperature range. For example, baking can be started approximately after 10 minutes, if the oil temperature at the bottom of the pan is kept at 250°C (Fig. 4).
4.2 Simulations of temperature profiles for cyclic baking

Once heat up is completed, baking of injera is started by pouring the dough on the surface of the pan. A single baking cycle includes: heat up, baking time and idle period. The baking period for all cyclic baking simulation is taken to be 150s. The idle period varies depending on baking pan thermal property, thickness and heated oil temperature.

Temperature profiles along the thickness of the ceramic baking pan and on the surface of the injera for different heated oil temperatures were simulated for five baking cycles. Node 149 represents the surface temperature of five injera (inj-1, inj-2…inj-5) during baking, and node 118 represents the surface temperature of the baking pan during heat up, baking, and idle periods (Fig. 5 & 6).

The idle periods were 165 and 120 seconds for heated oil temperatures of 250 and 275°C respectively. The injera temperature at the interface with the pan reaches the local boiling temperature, which is, 92°C in Addis Ababa. This has been proved experimentally. Generally, it takes approximately 30 minutes and 27 minutes to bake five injera for heated oil temperatures of 250 and 275°C respectively (Fig. 5 & 6). The higher portion of the supply energy is consumed during initial heat up. Hence, reducing idle or heat up time results in saving significant energy.

During the baking of injera, the temperature of baking pan and injera remain almost equal for a while. The temperature of the baking pan drops significantly during the start of baking (pouring the dough on the surface of the pan). The variation of temperature on the surface of baking pan and injera during baking of the first injera is shown as a magnified view (Fig. 7).

The effect of increasing baking pan thickness on heat up and baking period is also studied considering a 1cm thickness of ceramic baking pan. Five baking cycles were considered for heated oil temperature of 275°C (Fig. 8).

The heat up time increased from 355 seconds to 525 seconds due to the increase of baking pan size from 8mm to 10 mm. The idle period also increased from 120 to 145 seconds. The idle time (time required to heat up the baking pan to baking surface temperature) during the baking of five injera absorbs a significant amount of energy. Hence, reducing the thickness of baking pan, not only lowers the heat up time, but also reduces the idle period, which is very important for longer baking sessions.

On the other hand, large amount of energy is consumed during initial heat up. Hence, increasing the number of
injera baked per baking sessions is also one way of saving energy.

The temperature at the bottom and upper surfaces are almost identical (Fig. 9). Hence, it is reasonable to ignore the thickness of the steel at the bottom of the ceramic pan during the finite element mesh. Even though steel has short heat up time compared to ceramic, getting injera with good texture is a problem. In addition sticking of the steel surface to injera batter is another drawback of the steel pan. But, research is underway in our research group on developing non-sticking Aluminum pan using heated oil as a power source.

5. CONCLUSIONS

The Finite Element Method predicted well the temperature profiles during initial heat up and cyclic baking of solar powered injera baking pan. There is significant change in heat up time for different oil temperatures. From the simulation results it can be concluded that, the heat up time can be reduced by reducing thickness of the baking pan for a given supply oil temperature. This effect can be seen clearly by comparing the results for cyclic baking (Fig. 6 and 8). There is also significant reduction in idle period by reducing the thickness of the baking pan or by improving the supply oil temperature. Moreover, increasing the number of injera baked per baking session is one way of improving the energy efficiency of the system. There is significant consumption of energy during heat up compared to the overall baking session. Generally, the proposed solar powered baking pan gives reasonable heat up and baking time for 8mm thick ceramic pan with heated oil temperature of 275°C. Based on these findings, a new ceramic pan of 8 mm thick was manufactured in facilities of a local company and tested for baking with heated oil (Shell Thermia B). The overall solar powered injera baking system is currently under construction with the objective of delivering oil with 275°C at the kitchen.

6. NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>T</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>ΔT</td>
<td>Temperature change (°C)</td>
</tr>
<tr>
<td>(c_p)</td>
<td>Specific heat (J/kg.K)</td>
</tr>
<tr>
<td>D</td>
<td>Diffusion coefficient ((m^2/s))</td>
</tr>
<tr>
<td>(h_b)</td>
<td>Boiling coefficient (W/m²K)</td>
</tr>
<tr>
<td>(h_c)</td>
<td>Convection coefficient (W/m²K)</td>
</tr>
<tr>
<td>(h_r)</td>
<td>Radiative coefficient (W/m²K)</td>
</tr>
<tr>
<td>(h_{fg})</td>
<td>Latent heat (J/kg)</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity (W/m K)</td>
</tr>
<tr>
<td>(x_w)</td>
<td>Moisture content</td>
</tr>
</tbody>
</table>
Greek Letters

\( \varepsilon \) Ratio of vapor diffusion coefficient to the coefficient of total moisture diffusion

\( \delta \) Thermo-gradient coefficient \((J/kg)\)

\( \Gamma \) Boundary of the element

\( \rho \) Density \((kg/m^3)\)

Matrices and Vectors

\([K]\) Global stiffness matrix

\([C]\) Global capacitance matrix

\([Q]\) Global thermal load vector

Subscripts and Superscripts

\( \infty \) Surrounding (Ambient)

\( \text{Inj} \) Injera

\( s \) Surface

\( e \) Element

\( f \) Fluid

7. REFERENCES