ABSTRACT

The US Department of Energy has estimated that 280,000 MW of recyclable waste heat is expelled annually by U.S. industries. Further estimates suggest that harvesting it could result in a savings of $70 billion to $150 billion per year (1). Thus, any efficiency increase will result in savings to energy producers. Supercritical carbon dioxide (SCO₂) provides unique advantages over alternative waste heat recovery systems however, it also produces unique design challenges. We propose a novel energy recovery device based on a SCO₂ regenerative Rankine cycle for small-scale (1kW to 5kW) heat recovery. This study presents a thermodynamic SCO₂ cycle analysis for waste heat recovery from low temperature (200°C - 500°C) sources using small mass flow rates (20 – 60 grams/sec). This paper will present a prototype SCO₂ cycle architecture including details of key system components. Preliminary modeling suggests that SCO₂ systems are viable for low temperature waste heat recovery.

1. INTRODUCTION

SCO₂ has been considered a viable alternative working fluid for power cycles since the 1960s because it provides several advantages over steam and helium (2-6). The density of SCO₂ allows energy extraction devices to have a much smaller footprint than comparable steam and helium based turbo machinery (6). Additionally, the critical point of CO₂ is very low (31.1 °C and 7.4 MPa) compared to other fluids, allowing for heat transfer from low temperature (200 °C – 500 °C) sources to the supercritical state (7). Operating in the single supercritical phase throughout the proposed cycle reduces the need for two-phase hardware (6). However, due to the operating pressure and highly variable, non-linear fluid properties, suitable hardware for industrial use did not exist until recently (4, 5). Advancements in compact heat exchangers and turbo machinery coupled with the drive for business to become “green” has revived interest in SCO₂ power cycles leading to new solutions for energy addition and extraction (4, 5). The objective of this paper is to investigate the waste heat regenerative SCO₂ Rankine cycle performance and feasibility with low flow rate through mathematical modeling.

2. REGENERATIVE RANKINE CYCLE LAYOUT

The waste heat regenerative Rankine cycle is made up of six components as shown on the next page in Fig. 1. The SCO₂ cycle starts at a low side pressure above 7.5 MPa and a low side temperature of 35 °C, slightly above the critical point. After compression the SCO₂ is brought to the high-side pressure of 20 MPa and a temperature of 36 °C, approximately 1 °C higher than the pre-compressed state. An internal heat exchanger then heats the pressurized SCO₂ by exchange with low-pressure, post-expansion SCO₂. After exiting the internal heat exchanger the SCO₂ is heated in a second heat exchanger where addition is done via waste heat, raising the temperature to its ultimate value of approximately 200 °C. The heated and pressurized SCO₂ is
then expanded near isentropically to produce rotational energy. The rotational energy is converted to electricity by coupling the expander’s output shaft to a permanent magnet alternator. The fluid exits at a low-side pressure of 12.4 MPa and a temperature of 163 °C. The expanded fluid is then run through the internal heat exchanger where it is cooled by high-pressure, pre expansion SCO2. The cooled supercritical CO2 is finally passed through a radiator where it exits at the pre-compressed pressure and temperature.

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The three degrees of freedom considered for the parametric analysis are: volumetric expansion ratio, high side pressure, and the temperature drop across the cooler. In order to close the thermodynamic cycle the temperature drop across the cooler was assumed to be a function of all other parameters. A MATLAB program is used to vary the three degrees of freedom, discard any physically impossible cycles, then determine the most efficient parameter combination. The flow chart for this program is shown in Fig. 3.

To eliminate two degrees of freedom, the Carnot efficiency $E(1)$ dictates that the greatest efficiency will be achieved when $T_{cold}$ is minimized and $T_{hot}$ is maximized. Therefore, maxima and minima for these values are selected where $T_{cold}$ is above the supercritical region and $T_{hot}$ is lower than the waste heat source.

$$E(1) = \eta_{carnot} = 1 - \frac{T_{cold}}{T_{hot}}$$

Fig. 4 below is generated using the same MATLAB code and shows the dynamic relationship dictated by the boundary conditions. Using Fig. 4 the required temperature drop across the heat exchanger, cooler, and cycle efficiency may be determined for a given heat exchanger effectiveness.

Given conservative device efficiencies an overall cycle efficiency is found to be 11% with an optimum volumetric expansion ratio to be between 1.4 and 1.5. Furthermore, to achieve this cycle the heat exchanger need only be 78% effective.

**TABLE 1: OPERATING POINTS DEFINED BY CYCLE OPTIMIZATION.**

<table>
<thead>
<tr>
<th>State Points</th>
<th>T (K)</th>
<th>P (MPa)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$h$ (kJ/kg)</th>
<th>$s$ (kJ/kg-K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>308</td>
<td>12.4</td>
<td>776.61</td>
<td>278.19</td>
<td>1.23</td>
</tr>
<tr>
<td>2</td>
<td>319</td>
<td>20.0</td>
<td>810.40</td>
<td>289.38</td>
<td>1.24</td>
</tr>
<tr>
<td>3</td>
<td>417</td>
<td>20.0</td>
<td>338.83</td>
<td>513.32</td>
<td>1.86</td>
</tr>
<tr>
<td>4</td>
<td>473</td>
<td>20.0</td>
<td>258.82</td>
<td>597.79</td>
<td>2.05</td>
</tr>
<tr>
<td>5</td>
<td>436</td>
<td>12.4</td>
<td>176.78</td>
<td>577.23</td>
<td>2.08</td>
</tr>
<tr>
<td>6</td>
<td>328</td>
<td>12.4</td>
<td>537.76</td>
<td>353.29</td>
<td>1.47</td>
</tr>
</tbody>
</table>
Fig. 3: This flow chart depicts the optimization program used to determine the most efficiency cycle. Pressure index varies from 14 to 20 MPa with the upper bound of nP at 20 MPa. Similarly, the Volumetric Expansion Index varies from 1.1 to 2.45 with the upper bound of nV at 2.45.

Fig. 4: Plot generated from parametric analysis to determine cycle efficiency based on temperature drop over heat exchangers.

3.2 Heat Exchanger Sizing

Current heat exchanger analysis techniques call for evaluation of fluid properties using bulk average temperatures. However, this assumption is only valid for fluids with properties that vary linearly within the heat exchanger. Evaluating fluid properties at the bulk average temperature severely overestimates specific heat throughout the heat exchanger (7), as shown by the “Conventional” line in Fig. 5. Furthermore, there has been little research done to improve the analysis of such fluids other than computational simulations. To avoid overestimation, the heat exchangers were sized using a piecewise technique and the heat exchanger was split into dimensionless axial nodes where fluid properties varied more linearly as shown by the “Piecewise” line in Fig. 5.

Fig. 5: Conventional vs. Piecewise Method for heat exchanger analysis.
pump for the cycle. By comparing non-dimensional parameters such as the flow coefficient $E(4)$, head coefficient $E(5)$, and power coefficient $E(6)$ a suitable speed and characteristic diameter may be calculated using $E(2)$ and $E(3)$. Combining $E(4)$, $E(5)$, and $E(6)$ allows an efficiency to be specified per $E(7)$ (14, 15). Utilizing these results for an 85% efficiency yields a relatively small specific speed and a large specific diameter requirement. The Cordier diagram dictates that for this combination of operating points reciprocating pump is the optimal starting choice for the compression cycle (15). This determination was also repeated for the expander selection discussed later and shown in Fig. 8.

$$E(2) : N_s = \frac{N \sqrt{V_f}}{H^{3/4}}$$
$$E(3) : D_s = \frac{DH^{1/4}}{\sqrt{V_f}}$$
$$E(4) : \phi = \frac{V_f}{ND^3}$$
$$E(5) : \psi = \frac{H}{N^2D^5}$$
$$E(6) : \zeta = \frac{P}{\rho N^3 D^5}$$
$$E(7) : \eta = \frac{\phi \psi}{\zeta}$$

Where $N$ is rotations per minute, $V_f$ is volumetric flow rate of the fluid in cubic meters per second, $H$ is effective head in meters, $D$ is the effective diameter in meters, $P$ is power in Watts, and $\rho$ is density in kg/m$^3$.

There have been multiple attempts that address the optimization of a pump for $\text{SCO}_2$ applications (11-14) that indicate an efficiency of 85% to be reasonable and conservative. Efficiency may often be increased through later optimization, however, for the purpose of this paper an 85% efficiency is assumed to determine the work required to compress the $\text{SCO}_2$ from 12 MPa to 20 MPa.

Isentropic compression assumes ideal conditions with no losses. However, losses are incurred by leaks, heat transfer between the pump and fluid, and under or over compression leads to examination of the enthalpy difference between the compression state points. The compression cycle of $\text{SCO}_2$ occurs between state points 1 and 2, raising the pressure from 12 to 20 MPa. Work is determined as the enthalpy difference in the isentropic and polytropic compression of the fluid. This analysis yields an energy requirement of 0.2 kW to raise the pressure to the 20 MPa operating condition.

$$E(4) : W_R = \frac{[h(p_1, s_1) - h(p_2, s_2)]}{\eta}$$

4. COMPONENT SELECTION

4.1 Pump

$\text{SCO}_2$ systems present not only thermal design challenges but also gives rise to special considerations for the pump. In order to determine an initial pump design the concepts of the Cordier diagram are used determine an optimal type of
4.2 Expander

The SC\textsubscript{O}\textsubscript{2} in the expander undergoes near isentropic expansion in order to create mechanical energy to turn a permanent magnet alternator. The initial design was a toroidal engine with opposing pistons however, this architecture was found to be too difficult to fabricate. Therefore the expander design was revised into a more linear hexagonal variation. The latest iteration of the motor shown in Fig. 7 displays the current configuration of the motor ready to be rapid prototyped for a visual and mechanical inspection.

![Fig. 7: Cutaway of opposed piston hexagonal motor.](image)

This type of expander was chosen over a more conventional turbine expander due to low volumetric flow rates within the cycle and results from calculating the specific speed and specific diameter similar to the pump shown in Fig. 8.

![Fig. 8: Specific speed and specific diameter operating points for pump and expander plotted with efficiency curves for reciprocating pumps.](image)

The expander alternates intake and exhaust cycles to create axial piston movement. Each bank of three pistons is attached to a mounting plate that converts axial motion into rotational motion. A cam and a gear system is implemented to output power to a permanent magnet alternator. The design is still undergoing revision in order to optimize the expander for SC\textsubscript{O}\textsubscript{2}.

![Fig. 9: P-v diagram of the expander cycle.](image)

The cycle of the expander includes three optimal points: injection, expansion, and exhaust. Assumptions were made to neglect frictional losses, thermal losses through the expander itself, and steady state conditions. Fig. 9 shows the ideal expansion cycle of the expander from start to finish. The cycle begins with the piston at top dead center as SC\textsubscript{O}2 is injected into the expander at 20 MPa and 200°C. This raises the pressure from 12 to 20 MPa and the operating temperature of the previously expanded fluid from 90°C to 200°C within a relatively short amount of time. After the SC\textsubscript{O}2 is injected into the chamber the fluid is naturally allowed to expand, thus pushing the piston to bottom dead center. This expansion allows the pressure and temperature to drop while ideally maintaining constant enthalpy, and providing work for the system. The final leg of the cycle is a constant pressure evacuation of the piston chamber. This exhaust portion comes from the driving force done by the other side of the piston cycling through the expansion portion of the cycle. As the piston now slides back to top dead center, valves open inside the chamber allowing the SC\textsubscript{O}2 to evacuate back into the supply loop at reduced pressure and temperature of 12 MPa and 90°C. Once the expander has been built, careful testing will yield empirical data with which the theoretical results can be correlated.

4.3 Internal Heat Exchanger

The required heat transfer area from the heat exchanger analysis led to very large length requirements for a standard
counter flow concentric pipe configuration. Plate-based heat exchangers are desirable for large area requirements because of their large surface-area to volume ratio. Therefore, a compact plate heat exchanger architecture was chosen to reduce the overall size of the heat exchanger. The MATLAB code in section 3.2 was modified to arrive at a new area requirement using the plate convection correlation shown below in E(9) (10).

\[ E(5) : h = \frac{0.374k \text{Re}^{0.668} \text{Pr}^{0.33}}{D_h} \]

Due to the operating pressures and temperatures aluminum and copper heat transfer interfaces were not viable. The high side temperature is in the aging range of aluminum, which leads to eventual embrittlement and weakening below an acceptable level. Copper has acceptable temperature performance but, was not viable due to its low material strength. Therefore, stainless steel was selected because it provided the best compromise between strength and thermal conductivity at the operating pressure and temperature. Plate thickness was determined using the stress analysis equations shown below in E(10) (16).

\[ E(6) : \sigma_{\text{max}} = \frac{0.3078wb^2}{t^2} \]

The overall heat transfer coefficient was then corrected for conduction through the wall. The stainless steel thickness had little effect on the overall heat transfer coefficient because the thermal choke occurs in convection between the wall and the fluid core.

5. CONCLUSIONS

The above analysis examines the feasibility of a regenerative Rankine cycle utilizing \( \text{SCO}_2 \) for waste heat recovery in small-scale systems. Initial analysis suggests that recovery from low temperature sources approximately 200°C and above is feasible with an operating efficiency of 11% given the conservative operating parameters shown above in Table 1. Further analysis and experimental validation is required for optimal development of hardware that may be used for low flow rates. It is notable that the overall thermodynamic efficiency of the system is highly dependent on the internal heat exchanger effectiveness and it is expected that higher system efficiencies may be achieved after development and optimization of the system.

6. ACKNOWLEDGMENTS

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7. REFERENCES